A new perspective in power converters modelling

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A new perspective in power converters modelling: complementarity systems

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***Abstract*— Switched complementarity framework is proposed as an useful and simple way for modelling the dynamic behavior of power electronics converters. The voltage/current characteristic of each electronic device in each conducting state is assumed to be representable in piecewise linear form. It is shown how to obtain an equivalent circuit corresponding to the device characteristic, a complementarity representation for such characteristic and the switched complementarity model for the entire converter. The complementarity model is also shown to be effective for the time-stepping simulation of power converters.**

I. INTRODUCTION

Most power converters can be assumed to consist of linear elements (resistors, inductors, capacitors), voltage and current sources, and Electronic Devices (EDs) such as diodes and switches (thyristors, transistors, MOSFETs, etc.). A typical way for modelling power converters is to assume diodes and switches to be "ideal", to discriminate among the different modes of the converter and then to build for each mode a dynamic model [1]. The conditions for the commutation among the different modes will depend on the state (typically the voltage across the capacitors and the currents through the inductors), on external signals and on the control technique. However, this modelling approach, which also represents a necessary preliminary step for building averaged models, needs some knowledge on the way the converter operates, e.g. continuous or discontinuous conduction mode, and can become boring and tedious when the number of diodes and/or switches increases. Alternative modelling approaches such as maps [2] or PWM switch models [3] also suffer from the need of model structure modifications depending on the converter operating conditions. In this paper we propose the use of the complementarity formalism for modelling power converters. Complementarity models are characterized by a linear dynamic part and a suitable set of so called complementarity variables (interpreted as inputs and outputs of the dynamic part of the model), which are constrained not to be nonzero at the same time [4]. Complementarity systems [5] have been proposed as a framework for modelling (static) Resistors Diodes Sources (RDS) circuits, which includes only linear resistors, independent voltage and current sources and ideal diodes [6]. More recently, switched complementarity systems have been used to model switched electrical networks that contain Ideal Diodes (IDs) and Ideal Switches (ISs) [7].

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By using the complementarity framework the power converter can be modelled from a different perspective with respect to the classical power electronics approaches: the EDs are considered as external elements and modelled separately, then the entire model of the converter is obtained by integrating the EDs representations with the dynamic equations of the circuit. By assuming that the voltage/current behavior of each electronic device in each conducting state, say ON and OFF, can be represented by means of a piecewise linear voltage/current characteristic, it is shown how to obtain a corresponding RDS equivalent circuit by generalizing the procedure proposed in [8]. From such equivalent circuit it is simple to obtain a complementarity representation for the device characteristics. Finally, by including such representations into the converter dynamic model, the switched complementarity model of the entire converter can be constructed. The complementarity model is simple to be built, captures in a very simple way all modes of the converter and allows the idealization of the EDs characteristics at the desired level of abstraction. In order to obtain an efficient time-stepping simulation [9], the proposed models can be numerically integrated by exploiting already available algorithms for the integration of switched complementarity models [10]. Moreover, from a more analytical point of view, the complementarity model can be used to prove, by using a passivity concept, existence of solutions of the power converter model [11].

II. UNCONTROLLED ELECTRONIC DEVICES

Let assume that any (v, i) voltage/current characteristic, i.e. voltage on the horizontal axis and current on the vertical axis, of a diode or of a switch in a given conducting state, i.e. ON and OFF, can be approximated by a piecewise linear characteristic. In this section we show that any piecewise linear characteristic (also set–valued and nonmonotonic) can be represented in the following so called linear complementarity form:

$$\varphi = A_s\lambda + B_s z + g_s \tag{1a}$$
$$w = C_s\lambda + D_s z + h_s \tag{1b}$$
$$\mathbb{R}^p_+ \ni z \perp w \in \mathbb{R}^p_+, \tag{1c}$$

where the scalars φ and λ play the role of current and voltage or viceversa, (z, w) are nonnegative real vectors of length p and (z_i, w_i) denotes a pair of so called complementarity scalar variables. The (componentwise) interpretation of (1c) is that if two variables z_i and w_i are in a complementarity relation, then at least one of them must be zero and the other will be nonnegative.

A. Ideal diodes

Let us consider the representation of a diode characteristic reported in Fig. 1.

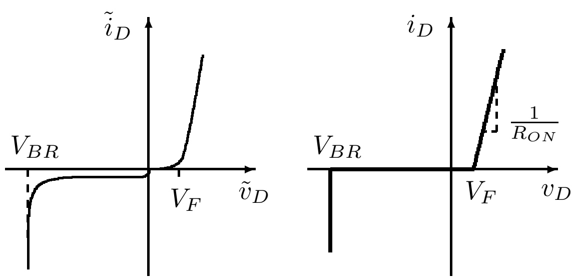

Fig. 1. Voltage/current characteristic of a diode and a corresponding idealized piecewise linear characteristic: V_{BR} is the breakdown voltage and V_F is the forward voltage.

Different approximated characteristics of the diode can be used, depending on the specific phenomena that one would either to consider or to neglect [12]. The diode characteristic is often idealized by introducing some simplifying assumptions: the piecewise linear characteristic (v_D, i_D) reported in Fig. 1 is an example for such possible approximation. By assuming $V_F = 0$, $R_{ON} = +\infty$ and $V_{BR} = -\infty$, the diode characteristic can be further simplified resulting in the (v_{ID}, i_{ID}) voltage/current characteristic reported in Fig. 2. Such approximated behavior of a power diode will be here called Ideal Diode (ID). The behavior of an ID can be described by

$$(-v_{ID} = 0 \wedge i_{ID} \in \mathbb{R}_+) \vee (-v_{ID} \in \mathbb{R}_+ \wedge i_{ID} = 0). \tag{2}$$

The relation (2) can be more compactly rewritten as the complementarity condition (1c) where z and w can be chosen as $z = i_{ID}$ and $w = -v_{ID}$ or conversely. Then, by choosing $\varphi = i_{ID}$ and $\lambda = v_{ID}$ the complementarity model (1) for the ID is complete, i.e. $p = 1$, $A_s = 0$, $B_s = 1$, $g_s = 0$, $C_s = -1$, $D_s = 0$ and $h_s = 0$.

Let us now assume that we want to represent the diode behavior by means of the piecewise linear characteristic

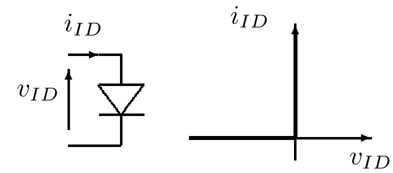

Fig. 2. Ideal Diode and the corresponding voltage/current characteristic.

(v_D, i_D) reported in Fig. 1. By analyzing the ID behavior it is possible to show that such characteristic corresponds to the equivalent circuit reported in Fig. 3. Then, by choosing $\varphi = v_D$, $\lambda = i_D$ and the complementarity variables $z_1 = -v_{ID_1}$, $z_2 = i_{ID_2}$, $w_1 = i_{ID_1}$ and $w_2 = -v_{ID_2}$ the model of such equivalent circuit (by applying the Kirchhoff rules and after some algebraic manipulations) can be represented in the form (1) with $p = 2$ and the following matrices

$$A_s = R_{\text{ON}},\ B_s = \begin{bmatrix}-1 & R_{\text{ON}}\end{bmatrix},\ g_s = V_F, \tag{3a}$$
$$C_s = \begin{bmatrix}1\\ R_{\text{ON}}\end{bmatrix},\ D_s = \begin{bmatrix}0 & 1\\ -1 & R_{\text{ON}}\end{bmatrix},\ h_s = \begin{bmatrix}0\\ V_F - V_{BR}\end{bmatrix}. \tag{3b}$$

Note that two pairs of complementarity variables are needed in order to represent the characteristic (v_D, i_D), whereas only one pair is enough for (v_{ID}, i_{ID}). The motivation for that is on the number of breaking points of the two characteristics. Moreover, different representations can be obtained with a different choice of the z and w variables.

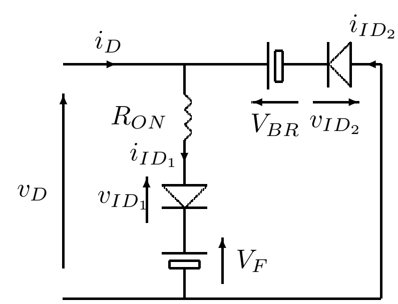

Fig. 3. Equivalent circuit corresponding to the (v_D, i_D) diode characteristic reported in Fig. 1.

The representation of an ID can be used to represent any piecewise linear characteristic in the complementarity form. In order to show that, we first show that any characteristic can be represented by means of an equivalent circuit that contains only resistors, independent sources and IDs, i.e. a so called RDS circuit. In particular, if the characteristic is nondecreasing only positive resistors are needed. Once the equivalent circuit has been obtained it is simple by using Kirchhoff rules and algebraic manipulations to obtain the complementarity representation (1).

B. Piecewise nondecreasing characteristics

We here reformulate and extend the procedure proposed in [8] for the construction of an RDS circuit corresponding to a given voltage/current characteristic. The basis of such procedure are the representations of strictly convex voltage/current and current/voltage characteristics. Let us consider the (v, i) voltage/current characteristic reported in Fig. 4, where G_i represent the admittance of the $(i+1)$-th linear part of the characteristic. The characteristics with the form reported in Fig. 4 will be indicated as convex characteristics. In Fig. 5 it is reported the corresponding equivalent circuit in canonical form, where

$$g_i = G_i - G_{i-1} \tag{4}$$

with $i = 0, \ldots, p$, being p the number of breaking points of the characteristic, and $G_{-1} = 0$.

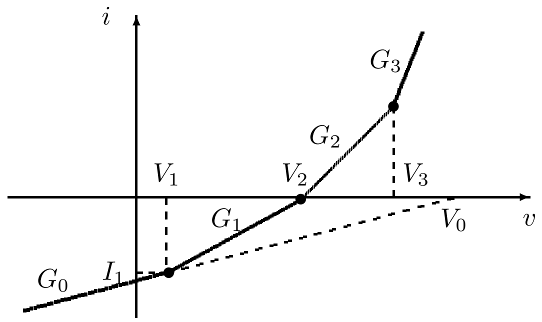

Fig. 4. Convex voltage/current piecewise linear characteristic with $p = 3$ breaking points.

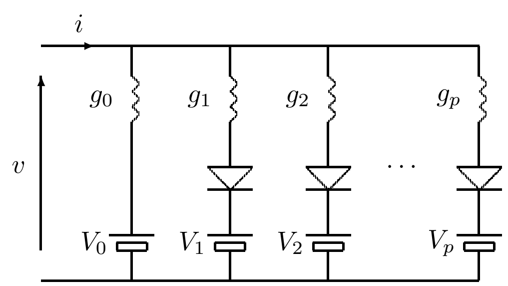

Fig. 5. Equivalent RDS circuit corresponding to convex voltage/current piecewise linear characteristics. To each ID a pair of complementarity variables can be associated.

For convex characteristics horizontal (vertical) parts are allowed only as the first (last) part of the characteristic. If there is an initial horizontal part it would be $G_0 = 0$ and $V_0 = +\infty$ which makes the first lag of the equivalent circuit not well defined. In that case the first lag of the equivalent circuit must be replaced by a current source whose value is the current value at the first breaking point (I_1 in Fig. 4). If the characteristic has a final vertical part, one must assume $g_p = +\infty$, i.e. a short circuit replacing the resistance, in the last lag of the equivalent circuit. The ID characteristic (see Fig. 2) can be simply verified to be a particular case of the situation reported in Fig. 4 ($p = 1$, $I_1 = V_1 = 0$, $g_1 = +\infty$).

Let us now consider the convex (i, v) current/voltage characteristic reported in Fig. 6, where R_i represent the resistance of the $(i+1)$-th linear part of the characteristic. In Fig. 7 it is reported the equivalent circuit in canonical form, where

$$r_i = R_i - R_{i-1} \tag{5}$$

with $i = 0, \ldots, p$, being p the number of breaking points of the characteristic, and $R_{-1} = 0$.

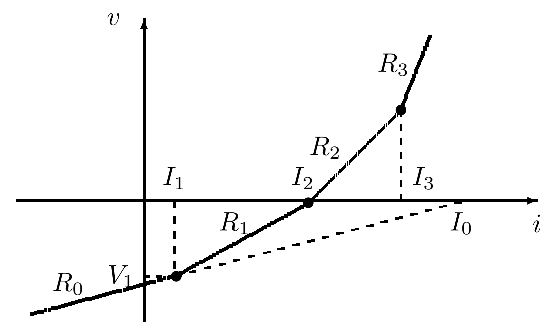

Fig. 6. Convex current/voltage piecewise linear characteristic with $p = 3$ breaking points.

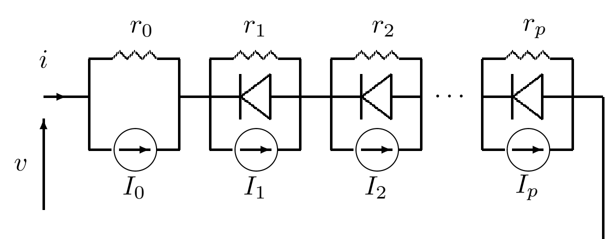

Fig. 7. Equivalent RDS circuit corresponding to convex current/voltage piecewise linear characteristics.

The current/voltage characteristic in Fig. 6 is assumed to be convex and therefore horizontal (vertical) parts are allowed only as the first (last) part of the characteristic. If there is an initial horizontal part it would be $r_0 = 0$ and $I_0 = +\infty$ which makes the first impedance of the equivalent circuit not well defined. In that case the first impedance of the equivalent circuit must be replaced by a voltage source whose value is the voltage value at the first breaking point (V_1 in Fig. 6). If the characteristic has a final vertical part, one must assume $r_p = +\infty$, i.e. an open circuit replacing the resistance in the last lag of the equivalent circuit.

It is now possible to construct a RDS equivalent circuit for a generic voltage/current piecewise linear characteristic, see Fig. 8. Without loss of generality, let assume that the first

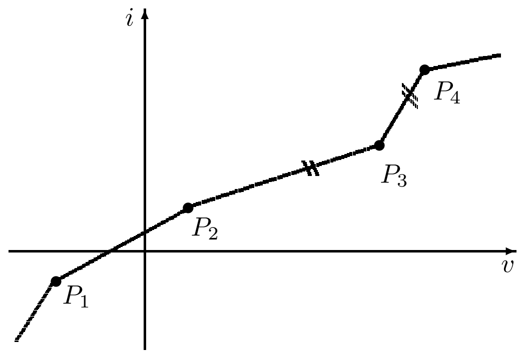

Fig. 8. A possible voltage/current piecewise linear characteristic with 4 breaking points P_i, $i = 1, \ldots, 4$. The first section (that includes P_1 and P_2, and finishes on P_3) is concave; the second section (that starts in P_2, includes P_3 and finishes on P_4) is convex and the third section (that starts in P_3 and includes P_4) is concave. Double strokes identify the segments of the characteristic common to concave and convex sections.

section of the voltage/current characteristic is concave, so as typical for electronic devices. The following steps must be carried out:

- Separate the entire characteristic into strictly concave or convex sections, see Fig. 8.
- For each section construct a new characteristic obtained by extending the last and first linear part of the original characteristic.
- Transform the concave sections characteristics obtained from the previous step into strictly convex characteristics by reversing the original axes.
- Subtract to each characteristic obtained from the previous step (except for the first one) the affine term corresponding to the first part of the characteristic, so that the resulting representation has a zero slope first part and the first breaking point has zero ordinate.
- Represent the new sections characteristics so as reported in Fig. 5 and Fig. 7. Moreover let denote by ξ_j the equivalent impedance of the j-th current/voltage convex characteristic with j odd numbers, and by y_k the equivalent admittance of the k-th voltage/current convex characteristic, with k even numbers.
- Construct the equivalent circuit so as reported in Fig. 9.

Analogous procedures can be simply defined using as basis the representations of strictly concave voltage/current and current/voltage characteristics and/or when the desired characteristic starts with a convex voltage/current section.

Following the procedure presented above it is straightforward to construct the equivalent circuits corresponding to any nondecreasing piecewise linear characteristic and from that the complementarity representation (1), e.g. the (v_D, i_D) characteristic reported in Fig. 1 and the corresponding equivalent circuit of Fig. 3. Other examples could be the following (possibly set-valued) characteristics: transistor, thyristor, Zener, step, max, min, saturation, dead-zone.

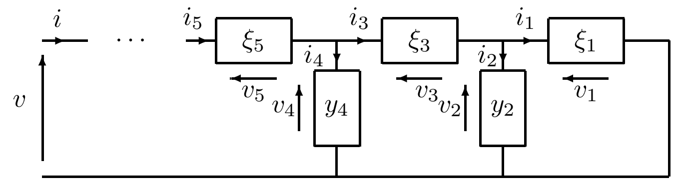

Fig. 9. Equivalent circuit corresponding to a voltage/current piecewise linear characteristic with $v_j = \xi_j(i_j)$ and $i_k = y_k(v_k)$. With reference Fig. 8 one has: $\xi_j = 0$ for any odd $j \geqslant 5$ and $y_k = 0$ for any even $k \geqslant 4$.

C. General piecewise linear characteristic

In the previous analysis only nondecreasing characteristics have been considered. If $\varphi(\lambda)$ is nonincreasing the representation (1) can be simply obtained by assuming $\varphi = \varphi^*$, $\lambda = -\lambda^*$ and representing the nondecreasing characteristic $\varphi^*(\lambda^*)$ in the complementarity form, following the procedure described above.

If the characteristic is not monotone with possible decreasing set-valued parts, a further manipulation is needed. Let assume that $\lambda \in \mathbb{R}$ and indicate by $-\bar{\gamma}$ with $0 \leqslant \bar{\gamma} < +\infty$ the most negative (finite) slope of the characteristic and by $-\bar{\delta}_i$ with $0 \leqslant \bar{\delta}_i < +\infty$, $i = 1, \ldots, J$ the (bounded) variations corresponding to the negative jumps occurring at $\lambda = \bar{\lambda}_i$, respectively. Then one can write

$$\varphi(\lambda) = -\bar{\gamma}\lambda - \sum_{i=1}^{J} \bar{\delta}_i step(\lambda - \bar{\lambda}_i) + \varphi^*(\lambda), \qquad (6)$$

where $\varphi^*(\lambda)$ will be nondecreasing. By using (6), the arguments presented above for the representations of $\varphi^*(\lambda)$ and the complementarity representation of the step function, the complementarity model (1) can be simply obtained.

III. Controlled electronic devices

In the previous analysis we have assumed that the electronic device was in a fixed conducting state. We now show that by generalizing the model (1) in the so called cone complementarity form, it is possible to represent the controlled electronic switches also when they are forced to change their conducting state. Let us first introduce the complementarity model of the Ideal Switch (IS). Let be v_{IS} the voltage across the switch, and i_{IS} the current through the switch. The behavior of an IS can be given by

$$\begin{aligned}&(\text{IS} \quad \text{is} \;\; \text{ON} \wedge -v_{IS} = 0 \wedge i_{IS} \in \mathbb{R})\\ &\qquad \vee (\text{IS} \quad \text{is} \;\; \text{OFF} \wedge -v_{IS} \in \mathbb{R} \wedge i_{IS} = 0). \qquad (7)\end{aligned}$$

The relations (7) can be rewritten in the following form

$$\mathcal{K}_\pi \ni z \perp w \in \mathcal{K}_\pi^* \tag{8}$$

where $z = -v_{IS}$, $w = i_{IS}$, $\pi = -1$ if IS is ON, $\pi = 1$ if IS is OFF, $\mathcal{K}_{-1} = \{0\}$, $\mathcal{K}_{-1}^* = \mathbb{R}$, $\mathcal{K}_1 = \mathbb{R}$, $\mathcal{K}_1^* = \{0\}$. The novelty with respect to the complementarity classification of the uncontrolled devices (so as the diode or a switch in a given state) is that the IS model includes the commutations by means of the switching function π which is time-varying. In particular the relation (8) is a generalization of (1c) with $p = 1$, which can be obtained by assuming $\pi = 0$ and $\mathcal{K}_0 = \mathcal{K}_0^* = \mathbb{R}_+$.

By using the IS model it is possible to model any electronic device (whose voltage/current behaviors in the different conducting state are representable by means of piecewise linear characteristics) in the following so-called cone complementarity form:

$$\varphi = A_s\lambda + B_s z + g_s \tag{9a}$$
$$w = C_s\lambda + D_s z + h_s \tag{9b}$$
$$\mathcal{C}_\pi \ni z \perp w \in \mathcal{C}_\pi^*, \tag{9c}$$

where φ and λ play the role of current and voltage or vice versa, and

$$\mathcal{K}_0 = \mathcal{K}_0^* = \mathbb{R}_+, \tag{10a}$$
$$\mathcal{K}_{-1} = \{0\}, \mathcal{K}_{-1}^* = \mathbb{R}, \tag{10b}$$
$$\mathcal{K}_1 = \mathbb{R}, \quad \mathcal{K}_1^* = \{0\}, \tag{10c}$$
$$\mathcal{C}_\pi = \mathcal{K}_{\pi_1} \times \mathcal{K}_{\pi_2} \times \cdots \times \mathcal{K}_{\pi_p}, \tag{10d}$$

with $\pi_i = 0$ constant if the corresponding complementarity variables (z_i, w_i) are associated to an ID, whereas π_i is the switching function if (z_i, w_i) are associated to an IS. The representations of both states of the switches (ON and OFF) are now included in (9). A simple way to show how it is possible to model a switch behavior is reported in Fig. 10, where the two ISs are complementary controlled, i.e. when the main switch corresponding to (v_S, i_S) is ON then IS_1 is ON ($\pi_1 = -1$) and IS_2 is OFF ($\pi_2 = -\pi_1 = +1$), and vice versa if the main switch is OFF. The equivalent circuits corresponding to the characteristics in the ON and OFF states are represented by the equivalent impedances Z_{ON} and Z_{OFF}, respectively, each of them representable as a RDS network. In other words, in this case (10d) becomes

$$\mathcal{C}_\pi = \mathcal{K}_{\pi_{IS1}} \times \mathcal{K}_{\pi_{IS2}} \times \mathcal{K}_0^p \tag{11}$$

where p is the total number of IDs included in the RDS representations of Z_{ON} and Z_{OFF}.

By exploiting the specific ED characteristics it is possible to obtain switches representations that involve a lower number of IDs and ISs with respect to the presented approach. Let us consider a switch given by the antiparallel connection of an ID and a MOSFET. If the switch is ON we have $\lambda_1 = v_S = 0$ and $\varphi_1 = i_S = \mathbb{R}$ (positive current flowing through the MOSFET and negative through the diode); if the switch is OFF one can write $\lambda_1 = v_S \in \mathbb{R}_+$ and $\varphi = i_S \in \mathbb{R}_-$ (the current can flow only thorugh the diode). The switch behavior can be represented by using a single pair of complementarity variables, i.e. $z_1 = -\varphi_1$, $w_1 = \lambda_1$ and $\mathcal{K}_{\pi_1} = \mathcal{K}_1 = \mathbb{R}$ if the switch is ON and $\mathcal{K}_{\pi_1} = \mathcal{K}_0 = \mathbb{R}_+$ if the switch is OFF.

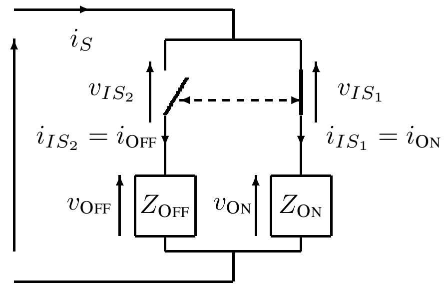

Fig. 10. Equivalent circuit for the complementarity representation of the characteristics (v_S, i_S) of a switch in both conducting states. The ISs conducting states reported in the figure correspond to the switch ON.

IV. COMPLEMENTARITY MODELS OF POWER CONVERTERS

In the previous analysis we have shown that the characteristic of any ED can be represented in the cone complementarity form (9) where (φ, λ) is the pair of voltage/current or current/voltage of the device and (z, w) is the pair of vectors of the complementarity variables associated to that device. In this section we show how, given the representations of the EDs included in a power converter, it is possible to obtain the representation of the entire converter.

In order to show the main idea that underpins the construction of the complementarity model of the entire power converter, let assume for a while to "put out" from the system all EDs and to consider current and voltage on each i-th ED as an input φ_i or as an output λ_i for the system. The circuit obtained by extracting m EDs, which will consist of linear elements and external sources, can be described by the state-space system

$$\dot{x} = A_d x + B_d \varphi + E_d u \tag{12a}$$
$$\lambda = C_d x + D_d \varphi + F_d u \tag{12b}$$

where x is the state vector, u denotes the external sources, φ and λ are vectors with m components, and (φ_i, λ_i) is the pair of voltage/current or current/voltage of the i-th ED. Note that possible state reduction, e.g. a three phase converter with currents equilibrium on the ac side, can be handled at this modelling stage following the typical approach used for classical state space representations of power electronics systems.

Each pair (φ_i, λ_i) can be represented in the form (9) corresponding to the specific piecewise liner characteristics chosen for that device. By grouping the representations of all devices the entire set of devices characteristics can be represented in the form (9), with vectors and matrices therein with suitable dimensions. So, putting together (12) and (9) one has

$$\dot{x} = A_d x + B_d \left[A_s \lambda + B_s z + g_s\right] + E_d u \tag{13a}$$

$$\lambda = C_d x + D_d \left[A_s \lambda + B_s z + g_s\right] + F_d u \tag{13b}$$

$$w = C_s \lambda + D_s z + h_s. \tag{13c}$$

By looking at (13b) if the matrix $D_d A_s$ has no eigenvalues in $+1$, the matrix $M \triangleq I - D_d A_s \in \mathbb{R}^{m \times m}$ is invertible and

$$\lambda = M^{-1}\left[C_d x + D_d B_s z + D_d g_s + F_d u\right]. \tag{14}$$

Now system (13) can be written in the following form:

$$\dot{x} = Ax + Bz + Eu + g \tag{15a}$$

$$w = Cx + Dz + Fu + h \tag{15b}$$

$$\mathcal{C}_\pi \ni z \perp w \in \mathcal{C}_\pi^* \tag{15c}$$

with

$$A := A_d + B_d A_s M^{-1} C_d, \tag{16a}$$

$$B := B_d A_s M^{-1} D_d B_s + B_d B_s, \tag{16b}$$

$$C := C_s M^{-1} C_d, \tag{16c}$$

$$D := D_s + C_s M^{-1} D_d B_s, \tag{16d}$$

$$E := B_d A_s M^{-1} F_d + E_d, \tag{16e}$$

$$F := C_s M^{-1} F_d, \tag{16f}$$

$$g := B_d \left[A_s M^{-1} D_d + I\right] g_s, \tag{16g}$$

$$h := h_s + C_s M^{-1} D_d g_s. \tag{16h}$$

Note that being M singular, it means that the converter structure has an algebraic loop not solvable and we get an ill-posed problem.

V. An Example: dc/dc Buck Converter

Let us consider the dc/dc buck converter reported in Fig. 11 where ED_1 and ED_2 are generic electronic devices. From the

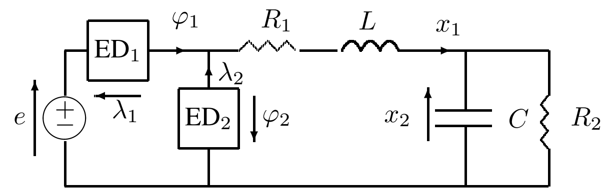

Fig. 11. Circuit scheme of a dc/dc buck converter.

circuit topology one obtains:

$$L\dot{x}_1 = -R_1 x_1 - x_2 - \varphi_2, \qquad C\dot{x}_2 = x_1 - \frac{1}{R_2} x_2. \tag{17}$$

Moreover, $x_1 = \varphi_1 + \lambda_2$, $\lambda_1 = \varphi_2 + e$. Assuming $u = e$ after some algebra the converter can be represented in the form (12a)-(12b) with

$$A_d = \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_2 C} \end{bmatrix}, \; B_d = \begin{bmatrix} 0 & -\frac{1}{L} \\ 0 & 0 \end{bmatrix}, \; E_d = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{18a}$$

$$C_d = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \; D_d = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; F_d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{18b}$$

Let assume that ED_2 is an ID. Then one can write

$$\varphi_2 = v_{ED_2} = -z_2, \quad \lambda_2 = i_{ED_2} = w_2, \tag{19a}$$

$$\mathcal{K}_0 \ni z_2 \perp w_2 \in \mathcal{K}_0^*. \tag{19b}$$

Let assume that ED_1 is an antiparallel connection of an ID and a MOSFET:

$$\varphi_1 = i_{ED_1} = -z_1, \quad \lambda_1 = v_{ED_1} = w_1, \tag{20a}$$

$$\mathcal{K}_{\pi_1} \ni z_1 \perp w_1 \in \mathcal{K}_{\pi_1}^*, \tag{20b}$$

where $\pi_1 = 1$ if the switch ED_1 is On and $\pi_1 = 0$ if the switch ED_1 is Off, see (10). Therefore the matrices of the model (9) are:

$$A_s = 0, \quad B_s = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad g_s = 0, \tag{21a}$$

$$C_s = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_s = 0, \quad h_s = 0, \tag{21b}$$

with $\mathcal{C}_\pi = \mathcal{K}_{\pi_1} \times \mathcal{K}_{\pi_2}$, $\mathcal{K}_{\pi_1} = \mathcal{K}_1 = \mathbb{R}$ if ED_1 is On and $\mathcal{K}_{\pi_1} = \mathcal{K}_0 = 0$ if ED_1 is Off, and $\mathcal{K}_{\pi_2} = \mathcal{K}_0 = \mathbb{R}_+$. Using (16) it is simple to achieve the complementarity representation (15) for the buck converter under investigation.

Let assume the following parameters: $e = 1$ V, $R_1 = 1\ \Omega$, $L = 3$ mH, $R_2 = 60\ \Omega$, $C = 250$ mF. Moreover an open loop pulse width modulation of ED_1 with a period equal to 200μs and a duty cycle equal to 0.3 is considered. A time-stepping simulation is carried out and at each time step it is solved the complementarity problem obtained by discretization of (15a) with the Euler method (sampling period of 1μs). The complementarity problem is solved by using the Lemke algorithm [10]. Simulation results are reported in Figs. 12–14. The time duration of the simulation (approx 20s) is almost the same of that achievable with Matlab/Simulink. However the Matlab model needs a finite state machine that represents the different modes of the converter, and that is quite complicated to be constructed also for this very simple configuration of power converter.

VI. Conclusions

Complementarity formalism have been shown to be an useful framework for modelling power converters. By using some examples of typical electronic devices and power converters topologies, we have illustrated the main advantages in using such methodology:

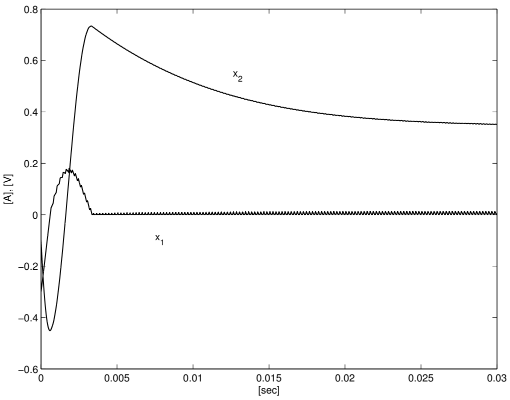

Fig. 12. Inductor current and capacitor voltage starting from $x_1(0) = -0.3$ A and $x_2(0) = -0.1$ V. At steady state the converter operates in discontinuous conduction mode.

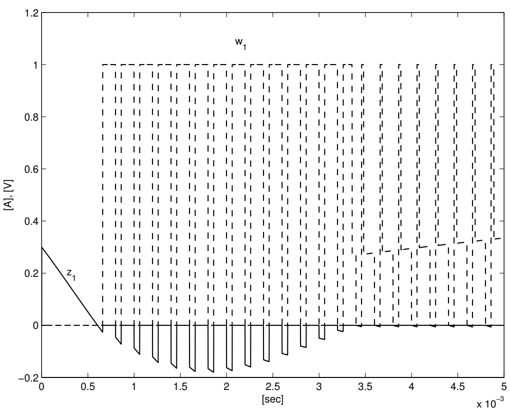

Fig. 13. Complementarity variables $z_1 = -\varphi_1$ (opposite of the current through ED_1) and $w_1 = \lambda_1$ (voltage on ED_1, dashed line).

- It is simple to build the model, without explicitly detail all modes of the converter.
- The equivalent piecewise linear voltage/current characteristics of the electronic devices can be fixed independently on the general model construction and so that some desired phenomena (f.i. breakdown, forward drops, leakage currents) are taken into account.
- Efficient algorithms for the numerical integration of switched complementarity models can be used for an accurate and fast simulation of power converters.

Moreover, in [11] the authors have shown that, by using the passivity concept, it is possible to prove existence and uniqueness for a class of complementarity systems that include power converters. Also, the proposed approach can be used for modelling other classes of nonlinear circuits, e.g. sensing circuits containing elements such as thermistor and bridges. On the other hand, although the complementarity formalism is valid also for controlled converters, it is not easy

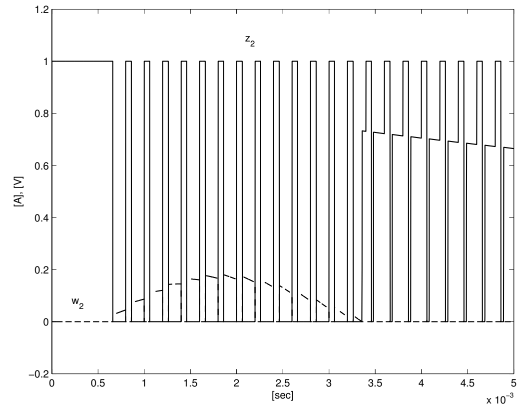

Fig. 14. Complementarity variables $z_2 = -\varphi_2$ (opposite of the voltage on ED_2) and $w_2 = \lambda_2$ (current through ED_2, dashed line).

to predict how the complementarity formalism might help for the power converters control design. That aspect, together with stability and frequency domain analysis are interesting directions for future research.

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